(JZS)Journal of Zankoy Sulaimani, Dec. 2005, 8(1) Part A(141-145)
گۆڤاری زانكۆی سلێمانی كانوونی یەكەم ٢٠٠٥ بەرگ ٨(١) بەش A (١٤١-١٤٥)

Sing le Machine Scheduling to Minimize Weighted Sum of Completion Times Added With the Maximum Tardiness - A Branch and Bound Approach

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Abstract

This paper considers the problem of scheduling n jobs on a single machine to minimize total weighted completion times and the maximum tardiness .A branch and bound algorithm is proposed to find optimal schedule. Our lower bound based on the late and early jobs. Computational experience on problems with up to 60 jobs for a special case and 50 jobs for a general case, where the previous works solve the problem up to 50 and 40 jobs for special and general case respectively .This indicates that the proposed algorithm is superior to other known algorithms .

Keywords: Single machine, weighted sum, Branch and bound.

1. Introduction

Several objectives can be optimized in a job-problem. The most classical is minimizing the makespan; that is, minimizing the completion time of the job that finishes last. This problem has been studied for many years and several custom approaches have been developed [1] .

In reality it's difficult to see problem to be solved by just one criterion. Most of the research in this area involves a single criterion ; however ,in reality operational effectiveness has many attributes including customer satisfaction, one time delivery ,work-in processing inventory, etc.

In order for scheduling to be in touch with reality, multi criteria problems must be studied .The simplest multi objective problems focus only on two criteria [2].

The problem of scheduling jobs on a single machine to minimize total weighted completion times and the maximum tardiness may be stated as follows:

Each job of the set N = {1, 2... n} is to be processed without interruption on a single machine which can handle only one job at a time. Job i($i \in N$)becomes available for processing at time zero, requires a positive integer processing time p_i and has a positive (real) weighted w_i.

2.Existing work

The single machine problem with this function has been studied extensively [1], and several custom approaches exist that use either branch and bound or dynamic programming [3].

Branch and bound method are enumeration techniques, which provide an approach to combinatorial optimization that applies to large class of problems. This method applied to minimize total weight completion time with dead lines [4], and to minimize total weighted completion time with release dates [5]. Also applied for many problems in scheduling [6].

3. Upper bound of the algorithm

We are driven to use heuristic methods, because many scheduling problem are NP-hard [7], so there are unlikely to be

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polynomial time algorithm for fined optimal solution.

The algorithm proposed in [8] is heuristic method for problem ($\sum_{i\in N} c_i + T_{max}$), since it doesn't guarantee optimality.

If $w_i \neq 1$, we treat with the general case which is ($\sum_{i\in N} w_i c_i + T_{max}$) problem.

The algorithm: ($w_i \neq 1$)

Step1:Find a schedule for $\sum_{i\in N} w_i c_i$ s.t. T_{max} is minimum[9].

Step2:For this schedule ,fix the first job in position 1 and let k=0.

Step3:Find a job from the remaining schedule with minimum p_j / w_j ; k = k+1.

Step4:If k = (n/2) for n even or , k = (n+1)/2 for n odd, goto step(6).

Step5:Goto step(3).

Step6:Compute $\sum_{i\in N} w_i c_i + T_{max}$ for the last schedule which is upper bound.

Numerical example

i	1	2	3
pi	5	7	3
di	2	5	10
wi	10	12	8

First we order the jobs in non-decreasing order of d_i (EDD) and T_{max}(EDD) = 7 .

Step 1: $R = \sum_{i=1}^{3} P_i = 15$, N= {1, 2, 3}, k=3 .

to find a job $j^* \ni pj^* \geq pj$ and $R - dj^* \leq T(EDD)$. Job 3 satisfies the condition , assign it in position 3 .

Now, $R = R - p_3 = 15-3 = 12$, job 2 satisfies the condition, assign it in position 2, and the last job in position 1 . So (1,2,3) is our schedule with $\sum_{i\in N} w_i c_i = 314$ s.t. $T_{max} = 7$.

Step 2: Fix job 1 in position 1 , k = 0

Step 3: Job 3 satisfies the condition , k= 1 .

Step 4: Since k≠ (n+1) / 2 , choose job 2

Step 5: We find a job 2 with minimum p_j / w_j ; k = 2.

Step 6: The sequence (1,3,2) is our upper bound with $\sum_{i\in N} w_i c_i + T_{max} = 304$

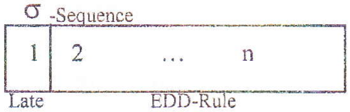
4. Derivation of the lower bound

The procedure which is used in this section to compute lower bound is based on partition the job into two parts , late jobs($c_i > d_i$)and early jobs ($c_i \leq d_i$). Very few studies deal with the problem of minimizing total earliness and tardiness penalties of multi-stage scheduling problem [10] and some studies describe methods of minimizing maximum earliness or tardiness for a flowshop.

We use this idea for one-machine problem to find a lower bound and describe the main components of the proposed enumerative algorithm based on a partition of the job N into two sets E and T with $N = E \cup T$, where E is the set of early jobs $E = \{i \in N / c_i \leq d_i\}$and T is the set of tardy jobs $T = \{i \in N / c_i > d_i\}$.

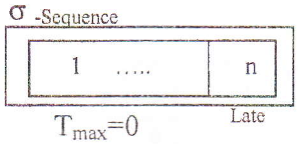
Let σ be the SPT sequence and $\sigma_{(i)}$ denotes the position of job i in the ordering σ .For this sequence we face three cases:

(i) If the first job is late $\sigma_{(1)}$ .Order the jobs in σ where $\sigma_{(i)} = \{\sigma_{(2)}, \sigma_{(3)}, ..., \sigma_{(n)}\}$ in EDD- rule

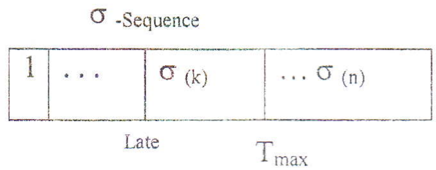


then compute T_{max} from the sequence .

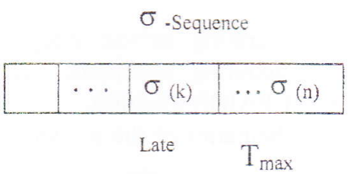
(ii) If the last job is late $\sigma_{(n)}$, means $\sigma_{(i)} = \{ \sigma_{(1)}, \sigma_{(2),\ldots,} \sigma_{(n-1)}\}$ is early and $T_{max} = T_{max}\,\sigma_{(n)}$.



(iii) If the first late job lies between $\sigma_{(1)}$ and $\sigma_{(n)}$,say $\sigma_{(k)}$ .Before $\sigma_{(k)}$ the jobs is early , order the jobs after $\sigma_{(k)}$ in EDD- rule .



So, in three cases we have a lower bound $\sum c_{i\,(SPT)} + T_{max}$, where T_{max} is one of the three cases .



Numerical example

i	1	2	3
pi	7	5	3
di	9	10	8

Order the jobs in SPT-rule and find c_i

i	3	2	1
pi	3	5	7
di	8	10	9
ci	3	8	15

3 Job 1 is late , since $T_1 = c_1 - d_1 > 0$ and $T_{max} = 6$. So $\sum c_{i\,(SPT)} + T_{max} = 26+6 = 32$ is lower bound .
i=1

5. Computational experience

The branch and bound algorithms were tested on $\sum w_i c_i + T_{max}$ problem with 10,20,30,40 and 50 jobs as
i∈N
well as on the $\sum c_i + T_{max}$ problem with 10,20,30,40,50,60 jobs. The algorithms run on CPU-PENTIUM 400
i∈N
MHZ , RAM 512 MB computer , using FORTRAN 90 compiler . Data were generated at random as follows :

For job i(i = 1,2,...,n) an integer processing time p_i generated form the uniform distribution [1,100] , an integer due-date d_i is generated form the uniform distribution [0, p_i] and an integer weight w_i from the uniform distribution [1,10] .

For every value of n , twenty problems of the two problems are generated .Table(1) shows the computation results of the $\sum c_i + T_{max}$ problem and Table (2) shows the results of the $\sum w_i c_i + T_{max}$ problem .
i∈N
i∈N

6. Conclusion and further works

We have developed a branch and bound algorithm, which is clearly superior to previous algorithms because it solves the problem until 60 jobs. Also provides a satisfactory method for solving small and medium sized problems.

Our results indicate that the two problems are slightly easier, and a valid lower bound is proposed to solve this multiple objective, since the problems were solved up to 60 jobs.

At a final conclusion, it should be stated that the multiple objective problems are difficult for a large n and some difficult cases of our problem have been arised .

We give the following topics for further investigations -Use this lower bound to another objective function.- Solve this problem with setup times.

Table (1): Results for the special case

N	AN	NS_1	NS_2	NS_3	NU
10	70.7	16	4	0	0
20	10.510	10	7	3	0
30	21.370	5	10	4	1
40	48.201	2	3	11	4
50	62.310	1	2	11	6
60	67.107	0	0	13	7

problem $\sum_{i\in N} c_i + T_{max}$ ....(1)

N : Number of the jobs .
AN: Average number of nodes in branch and bound search tree.
NS_1: Number of solved problems that require not more than 100 nodes .
NS_2: Number of solved problems that require not more than 1000 nodes .
NS_3: Number of solved problems that require over 1000 nodes .
NU: Number of unsolved problems when the limited of 50000 nodes is reached for problem (1) and 75000 for problem (2) .

Table (2): Results for the general case problem $\sum_{i\in N} w_i c_i + T_{max}$ ....(2)

N	AN	NS_1	NS_2	NS_3	NU
10	80	15	5	0	0
20	25.10	5	7	13	0
30	38.210	2	5	10	3
40	42.309	0	1	14	5
50	49.111	0	0	14	6

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نەخشاندنی یەك مەشین بۆ بچوك كردنەوەی سەرجەمی كێشی بۆ تەواوبوونی گشتی بەخستنە سەری گەورەترین دواكەوتن نزیك كردنەوەی پەلدار و سنوردار

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پوختە

لەم توێژینەوەیە باس لە كێشەی نەخشاندنی n لەبەرهەمەكانی یەك مەشینی دەكەین بۆ بچوك كردنەوەی سەرجەمی كێشی بۆ تەواوبوونی گشتی بەخستنە سەری گەورەترین دواكەوتن. خوارزمیەی پەلدار و سنووردارمان پێشكەش كرد بۆ دۆزینەوەی چاكترین شیكار . بچوكترین سنووردار كەبەكارمان هێناوە پشت ئەبەستێ بە بەرهەمە دواكەوتوو و پێشكەوتووەكان . وە ئەنجامە ژمێرياريەكانی كێشەكە گەیشتە (٦٠) بەرهەم بۆ بارودۆخە تايبەتيەكان و (٥٠) بەرهەم بۆ بارودۆخە گشتیەكان كاتێ كە كارە پێشووەكان لەسەر ئەم بابەتە گەیشتبووە (٥٠) بەرهەم بۆ بارودۆخە تايبەتەكان و (٤٠) بەرهەم بۆ بارودۆخە گشتیەكان . ئەمەش ئاماژە بەوە دەدات كە ئەم خوارزمیەی پێشكەشمان كراوە خێراترە لەوانی تر .

جدولة الماكنة الواحدة لتصغير المجموع الوزني للاتمام الكلي مضافا اليها اكبر تأخير- تقريب التفرع والتقييد

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الخلاصة

تناولنا في هذا البحث مسالة جدولة n من النتاجات على ماكنة واحدة لتصغير المجموع الوزني للاتمام الكلي مضافا اليها اكبر تاخير . قدمت خوارزمية التفرع والتقييد لايجاد الحل الامثل . القيد الادنى المستعمل تعتمد على النتاجات المتاخرة والمبكرة . النتائج الحسابية للمسائل وصلت الى ٦٠ نتاج للحالة الخاصة و ٥٠ نتاج للحالة العامة , بينما الاعمال السابقة وصلت الى ٥٠ نتاج للحالة الخاصة و ٤٠ نتاج للحالة العامة .وهذا مؤشر على ان الخوارزمية المقدمة اسرع من غيرها.

Received on19/10/2004 Accepted I n21/2/2005.
وەرگیرا لە ١٩/١٠/٢٠٠٤دا وەپەسەند کرا لە٢١/٢/ ٢٠٠٥ .